CLASS-12

Question Bank For Boards

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18/7/2018

"With detailed Solutions pertaining to each test paper."

Class 12 - Question Bank

A> Test No. 1, Test no. 8, Test No. 13, Test no. 17, } . Relations & Functions, Inverse Trigo, Matrices & Determinants, Binary Operations, LPP

B> Test no. 4, Test no. 7, Test no. 12, Test no. 16. } Differential Calculus

C> Test no. 3, Test no. 6, Test no. 14, Test no. 10, } Integral Calculus

D> Test no 2, Test no. 5, Test no. 11, Test no. 15, } Probability, Vectors, 3-D.

E> Test No. 209A, 209 B
310A, 310 B
411 A, 411 B
512A, 512 B
613 A, 613 B
714 A, 719 B
815 A, 815 B
916 A, 916 B } Questions of these test papers pertains to previous year papers of CBSE from 2009 - 2016. These questions have been bifurcated into set A and set B.
Set A → Ch. Relation & Functions + Algebra + Differentiation module + LPP.
Set B → Integration + Vectors & 3D + Prob

F> Test No. - C108 - 1
Test No - C310 - 1
Test No - C512 - 1
Test No - C815 - 1 } Full length Test Papers of old pattern containing 26 questions.

G> Test No - 1701
Test No - 1702
Test No - 1703 } Full length Test Paper pertaining to New pattern containing 29 questions.

H> Test Paper 1/17
17-15A, 17-15B,
17-16A, 17-16B } → Miscellaneous.

CONTENTS

→ Test No, 1701, 1702, 1703, 1/17,
→ Test No. 17-15 A, 17-15 B,
→ Test No. 17-16 A, 17-16 B,
→ Last Minute Handout.

will be given later.

TOTAL MARKS – 50

TIME – 100 mins.

TEST – 1

1) Show that function $f: R-\{3\} \to R-\{1\}$ given by $f(x) = \frac{4x+3}{6x-4}$ is a bijection. Find the inverse (4)

2) A binary operation * on set $\{0,1,2,3,4,5\}$ is defined as
$$a*b = \begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & a+b \geq 6 \end{cases}$$
Show that 0 is the identity element for this operation & each element $a (\neq 0)$ of the set is invertible with $6-a$, being the inverse of a. (4)

3) Show that relation S in set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a,b): a,b \in Z, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find set of elements related to 1 (4)

4) Express the following matrix as sum of symmetric & skew symmetric matrix & verify your result: $\begin{vmatrix} 3 & 4 & 2 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{vmatrix}$ (4)

5) i) If A is a square matrix of order 3 such that $|adj A| = 64$, Find $|A|$

ii) If A is a square matrix and $|A| = 2$, then write the value of $|AA^T|$

iii) For what values of x and y $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & y \end{bmatrix}$ is a skew symmetric matrix? (1+1+1)

6) Prove $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

or

$\begin{vmatrix} (b+c)^2 & ab & ac \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$ (6)

7) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ or (4)

Show that points $(a+5, a-4)$, $(a-2, a+3)$ & (a, a) do not lie on st. line for any value of a.

8) Prove $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$ (4)

9) Prove $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, $-\frac{1}{\sqrt{2}} \leq x \leq 1$ (4)

10) Show $\tan\left(\frac{1}{2}\sin^{-1} 3/4\right) = \frac{4-\sqrt{7}}{3}$ (4)

11) Solve: $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$ (4)

OR

$\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \pi/4$

12) Prove $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ (4)

OR

i) Find adj (adj A) where A = 3X3 matrix , $|A|=3$

ii) $|adj\,(adj\,A)|$ where A = 3X3 matrix , $|A|=2$

iii) Let A & B are symmetric matrices of same order then show 1+1+1+1

a) AB − BA is skew symmetric matrix

b) AB + BA " symmetric matrix.

13) $f\circ g\,(-3/2) + g\circ f\,(4/3)$ where $f(x)=[x]$, $g(x)=|x|$ (1)

SOLUTIONS TO EXAM-1

1) **For One-one:** let $x_1, x_2 \in A$. where $A = R - \{3\}$
$B = R - \{1\}$

let $f(x_1) = f(x_2)$

$\Rightarrow \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4} \Rightarrow (4x_1+3)(6x_2-4) = (4x_2+3)(6x_1-4)$

$\Rightarrow 24x_1x_2 + 18x_2 - 16x_1 - 12 = 24x_1x_2 + 18x_1 - 16x_2 - 12$

$\Rightarrow -34x_1 = -34x_2 \Rightarrow x_1 = x_2$

$\because f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in A$

$\therefore$ Function is one-one.

For onto: In order to prove range = codomain, we have to prove $f[g(y)] = y$ where g is f^{-1}.

let $y = \frac{4x+3}{6x-4} \Rightarrow 6xy - 4y = 4x+3 \Rightarrow 6xy - 4x = 3+4y$

$\Rightarrow 2x(3y-2) = 3+4y \Rightarrow x = \frac{3+4y}{6y-4} = g(y)$

$\therefore f(g(y)) = \frac{4g(y)+3}{6g(y)-4} = \frac{4 \times \frac{3+4y}{6y-4} + 3}{6 \times \frac{3+4y}{6y-4} - 4}$

$= \frac{\cancel{12}+16y+18y-\cancel{12}}{18+\cancel{24y}-\cancel{24y}+16} = \frac{34y}{34} = y$ $\therefore$ Function is bijective (proved).

$g(y) = f^{-1}(y) = \frac{3+4y}{6y-4} \Rightarrow f^{-1}(x) = \frac{3+4x}{6x-4}$ Ans

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2)

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$a * b = a+b$ if $a+b < 6$
$= a+b-6$ if $a+b \geq 6$

i) From the table we notice that $a*0 = 0*a = a$ (where $a \in$ set)
$\therefore 0$ is identity element.

ii) If $a*a' = 0$ then a' is inverse of a. $a*(6-a) = (\cancel{a}+6-\cancel{a}) - 6 = 0$. $\therefore 6-a$ is inverse of a.

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3) Given set $A = \{x \in Z : 0 \leq x \leq 12\}$; $S = \{(a,b) : a, b \in Z, |a-b|$ is divisible by 4$\}$

For reflexive: for $a \in A$, $(a,a) \in S \Rightarrow |a-a|$ is divisible by 4
$\Rightarrow 0$ is divisible by 4, true $\Rightarrow$ Hence reflexive.

For symmetric $\Rightarrow$ For $a, b \in A$, $(a,b) \in S \Rightarrow |a-b|$ is divisible by 4
$\Rightarrow |b-a|$ is divisible by 4. Hence $(a,b) \in S \Rightarrow (b,a) \in S \Rightarrow$ Symmetric

For transitive : $a, b, c \in A$

$(a,b) \in S$, $(b,c) \in S \Rightarrow |a-b|$ is divisible by 4 & $|b-c|$ is divisible by 4. $\therefore |a-c| = |(a-b)+(b-c)|$ is also divisible by 4 $\Rightarrow (a,c) \in S$. Hence transitive

$\therefore$ Relation S is equivalent proved.

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4) $A^T = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$ $\quad A+A^T = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix}$

$A - A^T = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$P = \frac{1}{2}(A+A^T) = \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix}$; $Q = \frac{1}{2}\left[A - A^T\right] \Rightarrow Q = \frac{1}{2}\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$\therefore P+Q = \frac{1}{2}\left[(A+A^T) + (A-A^T)\right] = \frac{1}{2}\begin{bmatrix} 6 & 4 & 6 \\ 8 & 10 & 6 \\ 4 & 6 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} = A$

$\therefore A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$ proved.

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5) i) $|\text{adj} A| = |A|^{n-1} \Rightarrow 64 = |A|^{3-1} \Rightarrow |A| = 8$

ii) $|A| = 2$, $|A^T| = 2$, $|A \cdot A^T| = |A| \cdot |A^T| = 2 \times 2 = 4$.

iii) For skew symmetric matrix $a_{ij} = -a_{ji}$ & diagonal elements are 0. $\therefore x = 2$ & $y = 0$.

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6) LHS : $\frac{1}{abc}\begin{vmatrix} -abc & a(b^2+bc) & a(c^2+bc) \\ b(a^2+ac) & -abc & b(c^2+ac) \\ c(a^2+ab) & c(b^2+ab) & -abc \end{vmatrix}$ $\quad R_1 = R_1 \times a$, $R_2 = R_2 \times b$, $R_3 = R_3 \times c$

$= \frac{1}{\cancel{abc}}\cancel{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$ Taking a, b, c common from each column

$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$ $\quad R_1 = R_1 + R_2 + R_3$

$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$

$= (ab+bc+ca)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ac+bc+ab) & 0 \\ ac+bc & 0 & -(ab+ac+bc) \end{vmatrix}$ $\quad C_2 = C_2 - C_1$, $C_3 = C_3 - C_1$

$= (ab+bc+ca)^3$ proved

$$\begin{vmatrix} (b+c)^2 & ab & ac \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} \overset{\text{OR}}{=} \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & a^2c \\ ab^2 & b(a+c)^2 & b^2c \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$$ Multiplying each row by a, b, c

$$= \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$ Taking a, b, c common from each column

$$= \begin{vmatrix} (b+c)^2-a^2 & 0 & a^2 \\ b^2-(a+c)^2 & (a+c)^2-b^2 & b^2 \\ 0 & c^2-(a+b)^2 & (a+b)^2 \end{vmatrix} = (a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ b-a-c & a+c-b & b^2 \\ 0 & c-a-b & (a+b)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$ Taking $(a+b+c)$ common from C_1 & C_2

$$= (a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & a+c-b & b^2 \\ 2a-2b & -2a & 2ab \end{vmatrix} = (a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

$R_3 \to R_3 - (R_1+R_2)$

$$= (a+b+c)^2\begin{vmatrix} b+c & a^2/b & a^2 \\ \frac{b^2}{a} & a+c & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$ $C_1 \to C_1 + \frac{C_3}{a}$, $C_2 \to C_2 + \frac{C_3}{b}$

$$= (a+b+c)^2\left\{2ab(b+c)(a+c) - \frac{b^2}{a}\times\frac{a^2}{b}\right\}$$

$$= (a+b+c)^2\,2ab\left(ba+bc+ac+c^2-\cancel{ab}\right) \Rightarrow (a+b+c)^2\,2abc \text{ proved.}$$

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7) $$\begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$ Multiplying each row by a, b, c

$$= \frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$ Taking abc common from 3rd column $$= -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$ proved.

OR

~~If the~~ $$\begin{vmatrix} a+5 & a-4 & 1 \\ a-2 & a+3 & 1 \\ a & a & 1 \end{vmatrix} = \begin{vmatrix} a+5 & a-4 & 1 \\ -7 & -7 & 0 \\ -5 & 4 & 0 \end{vmatrix}$$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= 1(-7\times 4 - 35) = -63 \neq 0.$

$\therefore$ three points are not collinear.

Note. If (x_1,y_1), (x_2,y_2), (x_3,y_3) are not collinear, then $\Delta = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \neq 0 \Rightarrow$ In other words 3 points represents a triangle & Δ represents the area of triangle

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8) $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

$$= \frac{1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} + \frac{1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1+\tan\left(\frac{1}{2}\cos^{-1}a/b\right)}$$

$$= \frac{\left[1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]^2 + \left[1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]^2}{1-\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} = \frac{2\left[1+\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]}{1-\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} \quad \text{—(i)}$$

We know $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ ~~$\Rightarrow \frac{2}{\tan\theta} =$~~

$\therefore$ (i) becomes $\dfrac{2}{\dfrac{1-\tan^2(\frac{1}{2}\cos^{-1}(a/b))}{1+\tan^2(\frac{1}{2}\cos^{-1}(a/b))}} = \dfrac{2}{\cos\left[2\times\frac{1}{2}\cos^{-1}\frac{b}{a}\right]} = \dfrac{2a}{b}$ proved

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9) $x = \cos 2\theta$; $\because -\frac{1}{\sqrt{2}} \le x \le 1 \Rightarrow -\frac{1}{\sqrt{2}} \le \cos 2\theta \le 1 \Rightarrow \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) \ge 2\theta \ge \cos^{-1}1 \Rightarrow 0 \le 2\theta \le 3\pi/4$

$\Rightarrow 0 \le \theta \le 3\pi/8 \Rightarrow \theta$ is in 1st quadrant.

$$\tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right] = \tan^{-1}\left[\frac{\sqrt{2}\{|\cos\theta| - |\sin\theta|\}}{\sqrt{2}\{|\cos\theta| + |\sin\theta|\}}\right] = \tan^{-1}\left[\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] = \tan^{-1}\left[\frac{\tan\pi/4-\tan\theta}{1+\tan\pi/4\cdot\tan\theta}\right] = \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)$$

$= \frac{\pi}{4} - \theta = \pi/4 - \frac{1}{2}\cos^{-1}x$ proved.

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10) $\tan\left(\frac{1}{2}\sin^{-1}3/4\right) = \frac{4-\sqrt{7}}{3}$; LHS : let $\frac{1}{2}\sin^{-1}3/4 = \theta$

$\Rightarrow 3/4 = \sin 2\theta \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = 3/4 \Rightarrow 3\tan^2\theta - 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta = \frac{8 \pm \sqrt{64-36}}{6} \Rightarrow \tan\theta = \frac{8\pm 2\sqrt{7}}{6} = \frac{4\pm\sqrt{7}}{3}$

~~$\theta =$~~ $\therefore \tan\theta = \frac{4+\sqrt{7}}{3}, \frac{4-\sqrt{7}}{3}$ $\quad \Rightarrow -\frac{\pi}{4} \le \theta \le \pi/4$

Now $-1 \le \sin 2\theta \le 1 \Rightarrow -\pi/2 \le 2\theta \le \pi/2$ $\quad \Rightarrow \tan\theta = \frac{4-\sqrt{7}}{3}$ proved.

$\therefore \tan\theta \not> 1$. $\therefore \tan\theta \ne \frac{4+\sqrt{7}}{3}$

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11) $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$

$\Rightarrow \tan^{-1}\left[\dfrac{\frac{x-1}{x+1}+\frac{2x-1}{2x+1}}{1-\frac{x-1}{x+1}\times\frac{2x-1}{2x+1}}\right] = \tan^{-1}\frac{23}{36}$

$\Rightarrow \tan^{-1}\left[\dfrac{2x^2-2x+x-1+2x^2-x+2x-1}{2x^2+3x+1-2x^2+3x-1}\right] = \tan^{-1}\frac{23}{36} \Rightarrow \frac{2x^2-1}{3x} = \frac{23}{36}$

$\Rightarrow \tan^{-1}\frac{2x^2-1}{3x} = \tan^{-1}\frac{23}{36}$ $\quad \Rightarrow 24x^2 - 32x + 9x - 12 = 0$

$\Rightarrow 24x^2 - 23x - 12 = 0 \Rightarrow x = -3/8$ or $4/3$

$\Rightarrow (8x+3)(3x-4) = 0$

Also $\frac{(x-1)(2x-1)}{(x+1)(2x+1)} < 1$

$\Rightarrow \frac{(x-1)(2x-1)}{(x+1)(2x+1)} - 1 < 0$

$\Rightarrow -\frac{6x}{(x+1)(2x+1)} < 0$

$+$ $\quad -1 \quad$ $+$ $\quad -1/2 \quad$ $-$ $\quad 0 \quad$ $+$

$\therefore$ $\boxed{x = 4/3}$ Ans.

OR

$\tan^{-1}\left(\dfrac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x^2-1}{x^2-4}}\right) = \pi/4 \Rightarrow \dfrac{(x-1)(x+2)+(x+1)(x-2)}{x^2-4-x^2+1} = \tan\pi/4$

$\Rightarrow x^2+x-2+x^2-x-2 = -3 \Rightarrow 2x^2 - 4 = -3$

$\Rightarrow (2x^2-1) = 0 \Rightarrow x = \pm 1/\sqrt{2}$

$\therefore$ $\boxed{x = \pm 1/\sqrt{2}}$ Ans

Also $\frac{x^2-1}{x^2-4} < 1 \Rightarrow \frac{x^2-1}{x^2-4} - 1 < 0$

$\Rightarrow \frac{3}{(x-2)(x+2)} < 0$

$+$ $\quad -2 \quad$ $-$ $\quad 2 \quad$ $+$

12> $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b+a^2b+b^3 & -a-a^3-b^2a & 1-a^2-b^2 \end{vmatrix}$ $C_1 \to C_1 - bC_3$, $C_2 \to C_2 + aC_3$

$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$ Taking $(1+a^2+b^2)$ common from C_1, C_2

$= (1+a^2+b^2)^2 \left\{(1-a^2-b^2+2a^2) - 2b(-b)\right\} = (1+a^2+b^2)^2 \left[1+a^2+b^2\right]$

$= \left[1+a^2+b^2\right]^3$ proved.

OR.

i> $adj(adj\,A) = |A|^{3-2} \cdot |A| = 2^1 \cdot |A| = 2 \cdot 2 = 4$

ii> $|adj(adj\,A)| = |A|^{(n-1)^2} = 2^{(3-1)^2} = 2^4 = 16$

iii> a) $(AB-BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T \cdot B^T = BA - AB$ $\left(\because \begin{matrix} A^T = A \\ B^T = B \end{matrix}\right)$

$\therefore (AB-BA)^T = -[(AB)-(BA)]$ ← skew symm.

b> $(AB+BA)^T = (AB)^T + (BA)^T = B^T A^T + A^T B^T = BA + AB$.

$\therefore (AB+BA)^T = (AB+BA)$ ← Symm. matrix.

3> $fog(-3/2) + gof(4/3)$

$= \left[\left|-3/2\right|\right] + \left|\left[4/3\right]\right| = [3/2] + |1| = 1+1 = 2$ Ans